A Novel Distributed Bernoulli Filter with Adaptive Event-Triggered Communication

Litao Zheng, Yunze Cai
Department of Automation
Shanghai Jiao Tong University
Shanghai 200240, China
zhenglitao@sjtu.edu.cn; yzcai@sjtu.edu.cn

Feng Yang, Lihong Shi
School of Automation
Northwestern Polytechnical University
Xi'an 710072, China
yangfeng@nwpu.edu.cn; shilihong@mail.nwpu.edu.cn

***Abstract*—This paper addresses communication bandwidth reduction and energy efficiency enhancement of a peer-to-peer sensor network for distributed target detection and tracking. A distributed Bernoulli filter with event-triggered communication is developed where each node broadcasts only local posteriors that achieve significant information gain. Specifically, for the cases where the Bernoulli density is no-target or single-target, the corresponding event-triggered strategies are constructed, respectively, in which the information discrepancy is measured via the Jeffreys divergence, and the triggering threshold is determined by the local information confidence coefficient. In addition, the presented method is combined with flooding protocol for internode communication, and weighted conservative fusion approaches are used to fuse the target existence probabilities and spatial distributions. Finally, simulation results demonstrate the effectiveness and superiority of the proposed approach.**

***Index Terms*—Distributed fusion, Bernoulli filter, event-triggered, flooding, conservative fusion**

I. Introduction

Target tracking in cluttered environments has attracted an ever-increasing interest in many realms. On the basis of random finite set (RFS) theory [1], [2], Bernoulli filter [3]–[5] has been identified as an effective tool for joint detection and tracking of a single target. When a peer-to-peer sensor network where each sensor node can only communicate with its neighbors is considered, conservative fusion approaches [6] including geometric average (GA) fusion and arithmetic average (AA) fusion, that can handle unknown correlations between nodes, have gained much attention. For distributed averaging, either average consensus [7]–[9] or distributed flooding [10]–[13] protocols can be employed. Generally speaking, the latter enjoys faster convergence efficiencies at the cost of larger node memory costs compared to the former. For distributed networks, several conservative fusion-based consensus Bernoulli filters [14], [15] have been proposed, which achieve network agreement by converging to treat all local estimators equally. To accommodate various RFS densities, some fusion weight design methods based on sensor and environment information [16] as well as on local posterior density information [17] have been developed to improve the fusion accuracy. Further, a weighted conservative fusion approach [18] has been presented to fuse Bernoulli densities, which not only achieves excellent fusion performance, but also has the property of closed-form expressions.

In practice, sensor nodes in a peer-to-peer network are often battery powered. Thus, to save energy and prolong the network lifetime, it is necessary to reduce the information transmission between nodes. In the context of distributed RFS filters, a series of event-triggered (ET) strategies [19]–[24] have been successfully used to reduce the communication costs. More specifically, the ET consensus Bernoulli filter [19] employs Kullback-Leibler (KL) divergence and Hellinger distance to measure the discrepancy between Bernoulli RFS densities. For multi-target tracking, the ET consensus cardinalized probability hypothesis density filter [20] and ET consensus labeled multi-Bernoulli filters [21], [22] are successively proposed. In addition, some works [23], [24] on event triggering thresholds are also given to adapt well to dynamic environments. However, the above ET consensus RFS filters are incapable of distinguishing information of high quality from that of low quality, which may result in estimation accuracy degradation. In addition, these distributed filtering methods experience difficulties in the case of low detection probability environments, due to the use of GA fusion manner for fusing the posterior densities [25], [26].

Based on the above discussions, this paper proposes a novel ET distributed Bernoulli filter, where the adaptive ET strategy is constructed by posterior information quality and discrepancy factors. The sensor, environment, and filtering information are employed to determine the posterior quality, and the Jeffreys divergence is adopted to measure the discrepancy between posteriors. Then, the Bernoulli densities are fused through weighted GA (WGA) or weighted AA (WAA) fusion manners [18]. Besides, the flooding protocol is utilized to achieve network consistency with a finite number of communication steps. The Gaussian mixture (GM) implementations of the proposed approach are also given. Finally, simulation results demonstrate the outperformance of the proposed ET distributed Bernoulli filter.

The rest of this paper is organized as follows. Section II briefly introduces background on Bernoulli filter and weighted conservative fusion. The proposed distributed Bernoulli filter with adaptive ET communication is described in Section III. Section IV evaluates the performance of the proposed approach via simulation experiments. Finally, conclusions and future works are given in Section V.

II. BACKGROUND

A. Local Bernoulli Filter

Given a peer-to-peer sensor network made up of S sensors, in which each node $i \in \mathcal{S} = \{1, \cdots, S\}$ communicates only with its neighbors $\mathcal{S}^i$. For joint detection and tracking of a single target in detection uncertainty and cluttered environments, each node i at each time instant k obtains the following posterior density characterized by the Bernoulli RFS X via the Bernoulli filter [5]

$$f_k^i(X) = \begin{cases} 1 - r_k^i, & X = \emptyset, \\ r_k^i p_k^i(x), & X = \{x\}, \end{cases} \tag{1}$$

where x is the target state, r_k^i and $p_k^i(x)$ are the existence probability (EP) and spatial probability density function (PDF), respectively.

B. Weighted Conservative Fusion

Given the local Bernoulli density as in (1), the local information confidence coefficients for the empty set and singleton set cases can be respectively defined by [18]

$$\pi_{\emptyset,k}^i = p_{d,k}^i / \left[\lambda_{c,k}^i(x) + 1\right], \tag{2}$$

$$\pi_{x,k}^i = \left(\varepsilon_1 \mathrm{tr}\left[P_k^i(x)\right] + \varepsilon_2 \left[\lambda_{c,k}^i(x) + 1\right] / p_{d,k}^i\right)^{-1}, \tag{3}$$

where $p_{d,k}^i$ is the detection probability, $\lambda_{c,k}^i(x)$ is the clutter rate associated with x, $\mathrm{tr}[P_k^i(x)]$ is the trace of the covariance, and ε_1, ε_2 are the scalars. Accordingly, the fusion weights can be expressed as

$$w_k^i(X) = \begin{cases} w_{\emptyset,k}^i, & X = \emptyset, \\ w_{x,k}^i, & X = \{x\}, \end{cases}$$

where $w_{\emptyset,k}^i = \pi_{\emptyset,k}^i / \sum_{j \in \mathcal{S}} \pi_{\emptyset,k}^j$ and $w_{x,k}^i = \pi_{x,k}^i / \sum_{j \in \mathcal{S}} \pi_{x,k}^j$.

Given the fusing Bernoulli densities $f_k^i(X)$, $i \in \mathcal{S}$, their WGA and WAA fusion can be respectively expressed as

$$\bar{f}_k(X) = \frac{\prod_{i \in \mathcal{S}} \left[f_k^i(X)\right]^{w_k^i(X)}}{\int \prod_{i \in \mathcal{S}} \left[f_k^i(X)\right]^{w_k^i(X)} \delta X}, \tag{4}$$

$$\tilde{f}_k(X) = \frac{\sum_{i \in \mathcal{S}} w_k^i(X) f_k^i(X)}{\int \sum_{i \in \mathcal{S}} w_k^i(X) f_k^i(X) \delta X}. \tag{5}$$

From an information-theoretic viewpoint, the WGA and WAA fusion manners symmetrically minimize the weighted sum of KL divergences between the fusing densities and fused one [27]–[30]. Besides, they are both conservative fusion [6], Fréchet means [31], and Hölder means [26].

Then, by exploiting Lemma 1 of [18], the fused EP and spatial PDF gained by the WGA fusion can be respectively written as

$$\bar{r}_k = \frac{\int \prod_{i \in \mathcal{S}} \left[r_k^i p_k^i(x)\right]^{w_{x,k}^i} dx}{\prod_{i \in \mathcal{S}} \left(1 - r_k^i\right)^{w_{\emptyset,k}^i} + \int \prod_{i \in \mathcal{S}} \left[r_k^i p_k^i(x)\right]^{w_{x,k}^i} dx},$$

$$\bar{p}_k(x) = \frac{\prod_{i \in \mathcal{S}} \left[p_k^i(x)\right]^{w_{x,k}^i}}{\int \prod_{i \in \mathcal{S}} \left[p_k^i(x)\right]^{w_{x,k}^i} dx}, \tag{6}$$

while that obtained by the WAA fusion can be respectively written as

$$\tilde{r}_k = \frac{\sum_{i \in \mathcal{S}} w_{x,k}^i r_k^i}{1 - \sum_{i \in \mathcal{S}} w_{\emptyset,k}^i r_k^i + \sum_{i \in \mathcal{S}} w_{x,k}^i r^i},$$

$$\tilde{p}_k(x) = \frac{\sum_{i \in \mathcal{S}} w_{x,k}^i r_k^i p_k^i(x)}{\sum_{i \in \mathcal{S}} w_{x,k}^i r_k^i}. \tag{7}$$

III. PROPOSED ET DISTRIBUTED BERNOULLI FILTER

In the distributed Bernoulli filter given in [18], each node $i \in \mathcal{S}$ has to transmit its posterior to its neighbors at each time index k. However, for actual sensor networks, this operation may lead to significant energy consumption and reduce the network lifetime. Thus, it makes sense to reduce the communication cost as much as possible while guaranteeing the performance of target detection and tracking. To this end, a triggering strategy is added to each node, which only transmits posterior information that is worth being broadcast. In addition, the given method is combined with weighted conservative fusion and flooding protocol for distributed averaging.

A. Event-Triggered Scheme

Assume that node i at time k stores the reference posterior $\mathring{f}_{k-1}^i(X)$, which is the most recent transmitted information by this node, and let $\mathring{f}_k^i(X)$ denote the prediction of this reference posterior. Then node i will broadcast its local posterior $f_k^i(X)$ to its neighbors if and only if the distance between $f_k^i(X)$ and $\mathring{f}_k^i(X)$ is large enough. In the context of Bernoulli density, the discrepancy between two posteriors can be decomposed into the discrepancy between their empty set cases plus the discrepancy between their singleton set cases, i.e.,

$$D\left(f, \mathring{f}\right) = D_\emptyset\left(r, \mathring{r}\right) + D_x\left(rp, \mathring{r}\mathring{p}\right). \tag{8}$$

In the following, the Jeffreys divergence [32] is utilized to measure the discrepancy between two Bernoulli densities, as it has the distance property, that is

$$\begin{aligned} D_J\left(f \| \mathring{f}\right) &= D_{KL}\left(f \| \mathring{f}\right) + D_{KL}\left(\mathring{f} \| f\right) \\ &= D_\emptyset\left(r_k^i, \mathring{r}_k^i\right) + D_x\left(r_k^i p_k^i, \mathring{r}_k^i \mathring{p}_k^i\right), \end{aligned}$$

where $D_{KL}(f \| \mathring{f})$ is the KL divergence from the densities f to $\mathring{f}$, and

$$D_\emptyset\left(r_k^i, \mathring{r}_k^i\right) = \left(r_k^i - \mathring{r}_k^i\right) \log \frac{1 - \mathring{r}_k^i}{1 - r_k^i},$$

$$D_x\left(r_k^i p_k^i, \mathring{r}_k^i \mathring{p}_k^i\right) = \int \left[r_k^i p_k^i(x) - \mathring{r}_k^i \mathring{p}_k^i(x)\right] \log \frac{r_k^i p_k^i(x)}{\mathring{r}_k^i \mathring{p}_k^i(x)} dx.$$

Note that the Jeffreys divergence cannot be analytically calculated, so appropriate approximation methods [33] need to be adopted.

In principle, to make full use of local information, the local posterior with high quality should be assigned a small triggering threshold. Thus, a simple and reasonable choice for the triggering threshold can be written as

$$\tau(\pi) = \left(1 - e^{-\pi}\right) \underline{\tau} + e^{-\pi} \overline{\tau}, \tag{9}$$

where π denotes the confidence coefficient for the posterior, $\underline{\tau}$ and $\overline{\tau}$ are respectively the upper and lower bounds of the threshold. Then the triggering conditions for the no-target and single-target cases can be respectively expressed as

$$D_{\emptyset}\left(r_k^i, \mathring{r}_k^i\right) > \tau\left(\pi_{\emptyset,k}^i\right),\ D_x\left(r_k^i p_k^i, \mathring{r}_k^i \mathring{p}_k^i\right) > \tau\left(\pi_{x,k}^i\right).$$

B. Proposed Algorithm

To achieve complete network agreement, flooding protocol is an effective strategy. At each flooding step $l = 1, \cdots, L$, each node i shares with its neighbors the information that has not yet been broadcast. Let $\mathcal{S}_l^i$ denote the set of nodes whose information has been collected by node i after l communication steps, and $\grave{\mathcal{S}}_l^i = \mathcal{S}_l^i \backslash \mathcal{S}_{l-1}^i$ denote the set of newly collected nodes.

Suppose that node i at time k stores the reference posteriors $\mathring{f}_{k-1}^j(X)$, $j \in \mathcal{S}$, of the entire network, and obtains the predicted reference posterior $\mathring{f}_k^j(X)$. Further, the prediction of the reference confidence coefficients $\mathring{\pi}_{\emptyset,k-1}^j$, $\mathring{\pi}_{x,k-1}^j$ can be constructed as

$$\mathring{\pi}_{\emptyset,k}^j = \alpha \mathring{\pi}_{\emptyset,k-1}^j,\ \mathring{\pi}_{x,k}^j = \alpha \mathring{\pi}_{x,k-1}^j,\ j \in \mathcal{S}, \tag{10}$$

where $0 < \alpha < 1$ is the penalty coefficient.

Let $\dot{\mathcal{S}}_{k,l}^i$ denote the set of triggered nodes whose information is newly collected by node i at flooding step l, and $\mathring{\mathcal{S}}_{k,l}^i = \grave{\mathcal{S}}_l^i \backslash \dot{\mathcal{S}}_{k,l}^i$ denote the set of non-triggered nodes. Then the flooding on confidence coefficients can be written as

$$\begin{aligned}
\pi_{\emptyset,k,l}^i &= \pi_{\emptyset,k,l-1}^i + \sum_{j \in \grave{\mathcal{S}}_l^i} \hat{\pi}_{\emptyset,k}^i, \\
\pi_{x,k,l}^i &= \pi_{x,k,l-1}^i + \sum_{j \in \grave{\mathcal{S}}_l^i} \hat{\pi}_{x,k}^i,
\end{aligned} \tag{11}$$

where $\pi_{\emptyset,k,0}^i = \pi_{\emptyset,k}^i$, $\pi_{x,k,0}^i = \pi_{x,k}^i$, and

$$\hat{\pi}_{\emptyset,k}^j = \begin{cases} \pi_{\emptyset,k}^j, & j \in \dot{\mathcal{S}}_{k,l}^i \\ \mathring{\pi}_{\emptyset,k}^j, & j \in \mathring{\mathcal{S}}_{k,l}^i \end{cases},\ \hat{\pi}_{x,k}^j = \begin{cases} \pi_{x,k}^j, & j \in \dot{\mathcal{S}}_{k,l}^i \\ \mathring{\pi}_{x,k}^j, & j \in \mathring{\mathcal{S}}_{k,l}^i \end{cases}.$$

Accordingly, the fusion weights can be obtained by

$$\begin{aligned}
w_{\emptyset,k,l}^i &= \pi_{\emptyset,k,l-1}^i / \pi_{\emptyset,k,l}^i,\ \hat{w}_{\emptyset,k,l}^{i,j} = \hat{\pi}_{\emptyset,k}^j / \pi_{\emptyset,k,l}^i, \\
w_{x,k,l}^i &= \pi_{x,k,l-1}^i / \pi_{x,k,l}^i,\ \hat{w}_{x,k,l}^{i,j} = \hat{\pi}_{x,k}^j / \pi_{x,k,l}^i.
\end{aligned} \tag{12}$$

Let $r_{k,0}^i = r_k^i$, $p_{k,0}^i = p_k^i$, and

$$\hat{r}_k^j = \begin{cases} r_k^j, & j \in \dot{\mathcal{S}}_{k,l}^i \\ \mathring{r}_k^j, & j \in \mathring{\mathcal{S}}_{k,l}^i \end{cases},\ \hat{p}_k^j(x) = \begin{cases} p_k^j(x), & j \in \dot{\mathcal{S}}_{k,l}^i \\ \mathring{p}_k^j(x), & j \in \mathring{\mathcal{S}}_{k,l}^i \end{cases},$$

then the flooding on EPs and spatial PDFs obtained by WGA can be given by

$$\begin{aligned}
r_{k,l}^i &= \frac{\bar{r}_{x,k,l}^i}{\bar{r}_{\emptyset,k,l}^i + \bar{r}_{x,k,l}^i}, \\
p_{k,l}^i(x) &= \frac{\left[p_{k,l-1}^i(x)\right]^{w_{x,k,l}^i} \prod_{j \in \grave{\mathcal{S}}_l^i} \left[\hat{p}_k^j(x)\right]^{\hat{w}_{x,k,l}^{i,j}}}{\int \left[p_{k,l-1}^i(x)\right]^{w_{x,k,l}^i} \prod_{j \in \grave{\mathcal{S}}_l^i} \left[\hat{p}_k^j(x)\right]^{\hat{w}_{x,k,l}^{i,j}} dx},
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
\bar{r}_{\emptyset,k,l}^i &= \int \left(1 - r_{k,l-1}^i\right)^{w_{\emptyset,k,l}^i} \prod_{j \in \grave{\mathcal{S}}_l^i} \left(1 - \hat{r}_k^j\right)^{\hat{w}_{\emptyset,k,l}^{i,j}}, \\
\bar{r}_{x,k,l}^i &= \int \left[r_{k,l-1}^i p_{k,l-1}^i(x)\right]^{w_{x,k,l}^i} \prod_{j \in \grave{\mathcal{S}}_l^i} \left[\hat{r}_k^j \hat{p}_k^j(x)\right]^{\hat{w}_{x,k,l}^{i,j}} dx.
\end{aligned}$$

The flooding on EPs and spatial PDFs obtained by WAA can be given by

$$\begin{aligned}
r_{k,l}^i &= \frac{\tilde{r}_{x,k,l}^i}{\tilde{r}_{\emptyset,k,l}^i + \tilde{r}_{x,k,l}^i}, \\
p_{k,l}^i(x) &= \frac{w_{x,k,l}^i r_{k,l-1}^i p_{k,l-1}^i(x) + \sum_{j \in \grave{\mathcal{S}}_l^i} \hat{w}_{x,k,l}^{i,j} \hat{r}_k^j \hat{p}_k^j(x)}{w_{x,k,l}^i r_{k,l-1}^i + \sum_{j \in \grave{\mathcal{S}}_l^i} \hat{w}_{x,k,l}^{i,j} \hat{r}_k^j},
\end{aligned} \tag{14}$$

where

$$\begin{aligned}
\tilde{r}_{\emptyset,k,l}^i &= 1 - w_{\emptyset,k,l}^i r_{k,l-1}^i - \sum_{j \in \grave{\mathcal{S}}_l^i} \hat{w}_{\emptyset,k,l}^{i,j} \hat{r}_k^j, \\
\tilde{r}_{x,k,l}^i &= w_{x,k,l}^i r_{k,l-1}^i + \sum_{j \in \grave{\mathcal{S}}_l^i} \hat{w}_{x,k,l}^{i,j} \hat{r}_k^j.
\end{aligned}$$

By combining the above operations with the Bernoulli filter [5], the ET distributed Bernoulli filter via flooding is summarized in Table I. For computational efficiency, the spatial PDF are often represented as GM, i.e., $p_k^i(x) = \sum_{a=1}^{N^i} \alpha_{k,a}^i \mathcal{N}(x; \hat{x}_{k,a}^i, P_{k,a}^i)$, where $\mathcal{N}(x; \hat{x}, P)$ denotes the Gaussian PDF with mean $\hat{x}$ and covariance P. In this case, the fused results (13) can be approximated by (16)-(18) of [18], while that (14) can be rewritten as (19)-(21) of [18].

It is worth noting that compared to the ET consensus Bernoulli filter [19], the proposed ET distributed filter only needs to perform the triggering condition discrimination once, thus it can achieve higher computational efficiency. However, the proposed algorithm requires greater storage requirements due to the fact that each node have to store the reference posteriors of the entire network.

IV. Simulation

To evaluate the performance of the proposed ET algorithm, consider a region of interest given by $[0m, 5000m] \times [0m, 5000m]$. The target state is denoted as $x_k = [p_{x,k}, \dot{p}_{x,k}, p_{y,k}, \dot{p}_{y,k}]^T$, where $[p_{x,k}, p_{y,k}]^T$ and $[\dot{p}_{x,k}, \dot{p}_{y,k}]^T$ are respectively position and velocity. The target appears at $k = 10s$ and disappears at $k = 90s$. The target motion follows a nearly constant velocity model as follows

$$x_k = \begin{bmatrix} 1 & \Delta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 1 \end{bmatrix} x_{k-1} + \begin{bmatrix} \frac{\Delta^2}{2} & 0 \\ \Delta & 0 \\ 0 & \frac{\Delta^2}{2} \\ 0 & \Delta \end{bmatrix} w_{k-1},$$

where $\Delta = 1\mathrm{s}$ and the process noise $w_{k-1} \sim \mathcal{N}(w; 0, qI)$ with $q = 5\mathrm{m}^2/\mathrm{s}^3$.

TABLE I
ET DISTRIBUTED BERNOULLI FILTER VIA FLOODING (NODE i, TIME k)

Inputs: $f^i_{k-1|k-1}(X)$

Obtain $f^i_k(X)$ via (91)-(105) of [5]
Obtain $\pi^i_{\emptyset,k}$ and $\pi^i_{x,k}$ via (2)-(3)
Obtain the predicted reference posteriors $\mathring{f}^j_k(X)$ based on $\mathring{f}^j_{k-1}(X)$, $j \in \mathcal{S}$
Compute the reference confidence coefficients $\mathring{\pi}^j_{\emptyset,k}$ and $\mathring{\pi}^j_{x,k}$ via (10)
Set $r^i_{k,0} = r^i_k$, $p^i_{k,0}(x) = p^i_k(x)$, $\pi^i_{\emptyset,k,0} = \pi^i_{\emptyset,k}$ and $\pi^i_{x,k,0} = \pi^i_{x,k}$,
if $D_\emptyset(r^i_k, \mathring{r}^i_k) > \tau(\pi^i_{\emptyset,k})$ or $D_x(r^i_k p^i_k, \mathring{r}^i_k \mathring{p}^i_k) > \tau(\pi^i_{x,k})$
  Broadcast $\{r^i_k,\ p^i_k(x),\ \pi^i_{\emptyset,k},\ \pi^i_{x,k}\}$ to its neighbors
  Set $\mathring{r}^i_k = r^i_k$, $\mathring{p}^i_k(x) = p^i_k(x)$, $\mathring{\pi}^i_{\emptyset,k} = \pi^i_{\emptyset,k}$ and $\mathring{\pi}^i_{x,k} = \pi^i_{x,k}$
end if
for $l = 1 : L$
  Obtain $\{r^j_k,\ p^j_k(x),\ \pi^j_{\emptyset,k},\ \pi^j_{x,k}\}_{j \in \dot{\mathcal{S}}^i_{k,l}}$ from its neighbors
  Set $\mathring{r}^j_k = r^j_k$, $\mathring{p}^j_k(x) = p^j_k(x)$, $\mathring{\pi}^j_{\emptyset,k} = \pi^j_{\emptyset,k}$ and $\mathring{\pi}^j_{x,k} = \pi^j_{x,k}$, $j \in \dot{\mathcal{S}}^i_{k,l}$
  Compute $\pi^i_{\emptyset,k,l}$ and $\pi^i_{x,k,l}$ via (11)
  Compute $w^i_{\emptyset,k,l}$, $\hat{w}^{i,j}_{\emptyset,k,l}$, $w^i_{x,k,l}$ and $\hat{w}^{i,j}_{x,k,l}$ via (12)
  Compute $r^i_{k,l}$ and $p^i_{k,l}(x)$ via WGA (13) or WAA (14)
end for
Set $r^i_{k|k} = r^i_{k,L}$, $p^i_{k|k}(x) = p^i_{k,L}(x)$

Outputs: $f^i_{k|k}(X)$

Given a peer-to-peer sensor network composed of 20 nonlinear sensors, as depicted in Fig. 1. Each sensor is characterized by a nonlinear measurement equation

$$h^i_k(x_k) = \begin{bmatrix} \sqrt{\left(p_{x,k} - p^i_x\right)^2 + \left(p_{y,k} - p^i_y\right)^2} \\ \text{atan2}\left(p_{x,k} - p^i_x, p_{y,k} - p^i_y\right) \end{bmatrix},$$

with (p^i_x, p^i_y) is the position of the sensor i, and the nonlinear measurement noise covariance matrix has been set to $R^i = \text{diag}\left(\sigma^2_r, \sigma^2_\theta\right)$ with $\sigma_r = 10m$, $\sigma_\theta = 1°$. The clutter intensity has been equal to $\lambda^i_k = 10$. The target detection probability coefficient has been set to $p^i_d = 0.9$.

The proposed ET distributed filters based on WGA (ET-WGA) and WAA (ET-WAA) are compared with the randomly triggered (RT) distributed filters based on WGA (RT-WGA) and WAA (RT-WAA), as well as the local Bernoulli filter. Simulations including 200 Monte Carlo trials have been performed. The optimal subpattern assignment (OSPA) error [34] with cutoff $c = 50\text{m}$ and order $p = 2$ has been employed as the performance index.

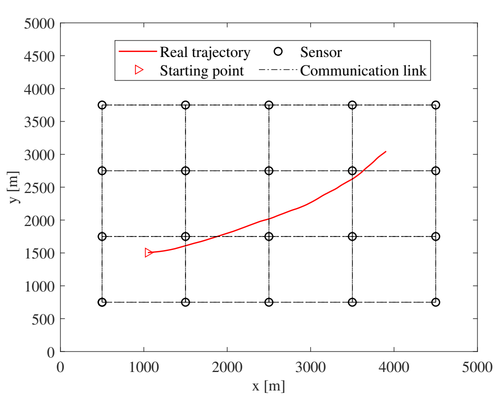

Fig. 1. Sensor network and target trajectory.

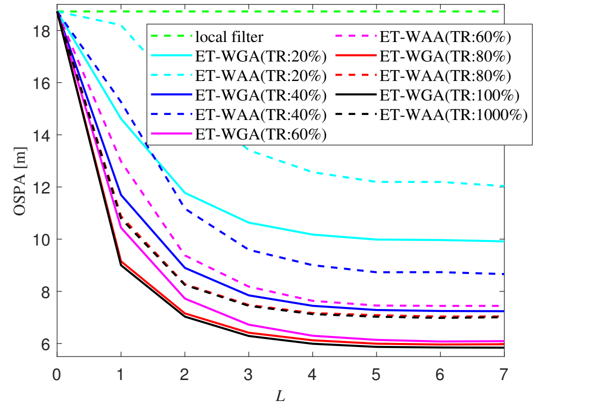

Fig. 2. OSPAs of the proposed filters under different TRs for different communication steps L.

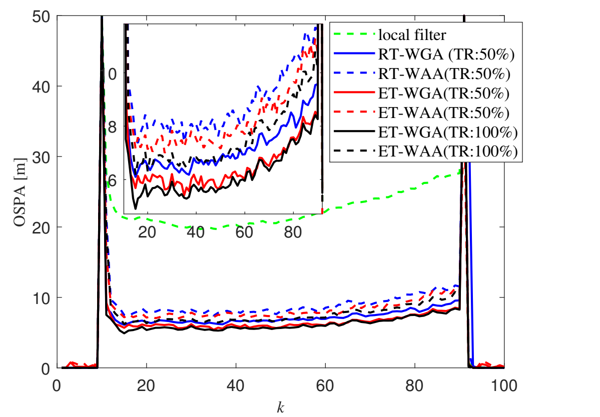

Fig. 3. OSPAs of the considered filters for different time steps k when the number of communication steps is equal to the network diameter.

Fig. 2 shows the OSPAs of the proposed ET filters under different triggering rates (TRs) averaged over all sensors and time steps versus different flooding steps L. Note that the proposed filter is used as a benchmark when the TR is 100%. It can be observed that as the number of iterations increases, the estimation accuracy of the proposed filters is improved. In particular, the estimation performance of the proposed algorithm is basically the same as the benchmark when the TR is 80%, which reflects the effectiveness of the proposed algorithms in terms of reducing the communication costs.

Fig. 3 shows the OSPAs of the considered filters over all sensors versus different time steps k. It can be seen that the proposed filters outperform the RT filters. In addition, when the TR is 50%, the proposed algorithm provides estimation accuracy degradation relative to the benchmark of 16.0% for $L = 1$, 5.1% for $L = 4$, and 4.3% for $L = 7$, which demonstrates the advantages of the proposed algorithm in large-scale networks.

V. Conclusion

This paper has presented an adaptive ET strategy for Bernoulli posterior density, which has been combined with distributed flooding protocol to derive WGA and WAA fusion-based ET distributed Bernoulli filters. The outstanding performance of the proposed filters has been demonstrated via simulation results in distributed target tracking scenarios. Possible future work will focus on extending the proposed approaches to unlabeled and labeled multi-Bernoulli densities [35], [36], and to likelihood consensus strategy [37].

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